

BONMIN Users' Manual

Pierre Bonami and Jon Lee

Version 1.7
Updated May 2013

1 Introduction

BONMIN (Basic Open-source Nonlinear Mixed INteger programming) is an open-source code for solving general MINLP (Mixed Integer NonLinear Programming) problems. It is distributed on **COIN-OR** (www.coin-or.org) under the CPL (Common Public License). The CPL is a license approved by the **OSI**¹, (Open Source Initiative), thus BONMIN is OSI Certified Open Source Software.

There are several algorithmic choices that can be selected with BONMIN. **B-BB** is a NLP-based branch-and-bound algorithm, **B-OA** is an outer-approximation decomposition algorithm, **B-iFP** is an iterated feasibility pump algorithm, **B-QG** is an implementation of Quesada and Grossmann's branch-and-cut algorithm, **B-Hyb** is a hybrid outer-approximation based branch-and-cut algorithm and **B-Ecp** is a variant of **B-QG** based on adding additional ECP cuts.

Some of the algorithmic choices require the ability to solve MILP (Mixed Integer Linear Programming) problems and NLP (NonLinear Programming) problems. The default solvers for these are, respectively, the COIN-OR codes **Cbc** and **Ipopt**. In turn, **Cbc** uses further COIN-OR modules: **Clp** (for LP (Linear Programming) problems), **Cgl** (for generating MILP cutting planes), as well as various other utilities. It is also possible to step outside the open-source realm and use **Cplex** as the MILP solver and **FilterSQP** as the NLP solver.

Additional documentation can be found on the **Bonmin** homepage at

<http://www.coin-or.org/Bonmin>

and wiki at

<https://projects.coin-or.org/Bonmin>

Types of problems solved

BONMIN solves MINLPs of the form

¹<http://www.opensource.org>

$$\begin{aligned}
& \min f(x) \\
& \text{s.t.} \\
& g^L \leq g(x) \leq g^U, \\
& x^L \leq x \leq x^U, \\
& x \in \mathbb{R}^n, x_i \in \mathbb{Z} \forall i \in I,
\end{aligned}$$

where the functions $f : \{x \in \mathbb{R}^n : x^L \leq x \leq x^U\} \rightarrow \mathbb{R}$ and $g : \{x \in \mathbb{R}^n : x^L \leq x \leq x^U\} \rightarrow \mathbb{R}^m$ are assumed to be twice continuously differentiable, and $I \subseteq \{1, \dots, n\}$. We emphasize that BONMIN treats problems that are cast in *minimization* form.

The different methods that BONMIN implements are exact algorithms when the functions f and g are convex but are only heuristics when this is not the case (i.e., BONMIN is not a *global* optimizer).

Algorithms

BONMIN implements six different algorithms for solving MINLPs:

- B-BB: a simple branch-and-bound algorithm based on solving a continuous nonlinear program at each node of the search tree and branching on variables [7]; we also allow the possibility of SOS (Type 1) branching
- B-OA: an outer-approximation based decomposition algorithm [6, 8]
- B-QG: an outer-approximation based branch-and-cut algorithm [11]
- B-Hyb: a hybrid outer-approximation / nonlinear programming based branch-and-cut algorithm [2]
- B-Ecp: another outer-approximation based branch-and-cut inspired by the settings described in [1]
- B-iFP: an iterated feasibility pump algorithm [3] .

In this manual, we will not go into a further description of these algorithms. Mathematical details of these algorithms and some details of their implementations can be found in [2] and [5] .

Whether or not you are interested in the details of the algorithms, you certainly want to know which one of these six algorithms you should choose to solve your particular problem. For convex MINLPs, experiments we have made on a reasonably large test set of problems point in favor of using B-Hyb (it solved the most of the problems in our test set in 3 hours of computing time). Nevertheless, there are cases where B-OA is much faster than B-Hyb and others where B-BB is interesting. B-QG and B-ECP correspond mainly to a specific parameter setting of B-Hyb but they can be faster in some case. B-iFP is more tailored at finding

quickly good solutions to very hard convex MINLP. For nonconvex MINLPs, we strongly recommend using B-BB (the outer-approximation algorithms have not been tailored to treat nonconvex problems at this point). Although even B-BB is only a heuristic for such problems, we have added several options to try and improve the quality of the solutions it provides (see Section 5.3). Because it is applicable to more classes problem B-BB is the default algorithm in BONMIN.

Required third party code

In order to run BONMIN, you have to download other external libraries (and pay attention to their licenses!):

- [Lapack](#) (Linear Algebra PACKage)
- [Blas](#) (Basic Linear Algebra Subroutines)
- a sparse linear solver that is supported by Ipopt, e.g., MA27 from the [HSL](#) (Harwell Subroutine Library), MUMPS, or Pardiso.

Note that Lapack and the Blas are free for commercial use from the [Netlib Repository](#)², but they are not OSI Certified Open Source Software. The linear solver MA27 is freely available for noncommercial use.

The above software is sufficient to run BONMIN as a stand-alone C++ code, but it does not provide a modeling language. For functionality from a modeling language, BONMIN can be invoked from [AMPL](#)³ (no extra installation is required provided that you have a licensed copy of AMPL installed), though you need the ASL (AMPL Solver Library) which is obtainable from the Netlib.

BONMIN can use FilterSQP [9] as an alternative to [Ipopt](#) for solving NLPs.

Also, in the outer approximation methods B-OA and B-iFP, some MILP problems are solved. By default BONMIN uses [Cbc](#) to solve them, but it can also be set up to use the commercial solver [Cplex](#)⁴.

2 Obtaining BONMIN

The BONMIN package consists of the source code for the BONMIN project but also source code from other [COIN-OR](#) projects:

- [BuildTools](#)
- [Cbc](#)
- [Cgl](#)
- [Clp](#)
- [CoinUtils](#)

²<http://www.netlib.org>

³<http://www.ampl.com>

⁴<http://www.cplex.com>

- [Ipopt](#)
- [Osi](#)

When downloading the BONMIN package you will download the source code for all these and libraries of problems to test the codes.

Before downloading BONMIN you need to know which branch of Bonmin you want to download. In particular you need to know if you want to download the latest version from:

- the Stable branch, or from
- the Released branch.

These different version are made according to the guidelines of COIN-OR. The interpretation of these guidelines for the Bonmin project is explained on the wiki pages of Bonmin.

The main distinction between the Stable and Release branch is that a stable version that we propose to download may evolve over time to include bug fixes while a released version will never change. The released versions present an advantage in particular if you want to make experiments which you want to be able to reproduce the stable version presents the advantage that it is less work for you to update in the event where we fix a bug.

The easiest way to obtain the released version is by downloading a compressed archive from [Bonmin archive directory](#).

The only way to obtain one of the stable versions is through [subversion](#).

In Unix⁵-like environments, to download the latest stable version of Bonmin (1.7) in a sub-directory, say `Bonmin-1.7` issue the following command

```
svn co https://projects.coin-or.org/svn/Bonmin/stable/1.7 Bonmin-1.7
```

This copies all the necessary COIN-OR files to compile BONMIN to `Bonmin-1.7`. To download BONMIN using svn on Windows, follow the instructions provided at [COIN-OR](#).

2.1 Obtaining required third party code

BONMIN needs a few external packages which are not included in the BONMIN package.

- Lapack (Linear Algebra PACKage)
- Blas (Basic Linear Algebra Subroutines)
- A sparse linear solver.
- Optionally ASL (the AMPL Solver Library), to be able to use BONMIN from AMPL.

⁵UNIX is a registered trademark of The Open Group.

Since these third-party software modules are released under licenses that are incompatible with the CPL, they cannot be included for distribution with BONMIN from COIN-OR, but you will find scripts to help you download them in the subdirectory `ThirdParty` of the BONMIN distribution. In most Linux distributions and CYGWIN, Lapack and Blas are available as prebuild binary packages in the distribution (and are probably already installed on your machine).

Linear solvers are used by Ipopt. The most up-to-date information regarding the supported linear solvers and how to install them is found in [Section 2.2](#) of the Ipopt manual.

Several options are available for linear solvers: MA27 from the Harwell Subroutine Library (and optionally, but strongly recommended, MC19 to enable automatic scaling in `Ipopt`), MA57 or Mumps. In our experiment MA27 and MA57 usually perform significantly better but they are freely available only for non-commercial, academic use. Note that linear solvers can also take advantage of Metis.

3 Installing BONMIN

The build process for BONMIN should be fairly automatic as it uses [GNU auto-tools](#). It has been successfully compiled and run on the following platforms:

- Linux using g++ version 4.5
- Windows using version Cygwin 1.5.18
- Mac OS X using gcc 4.5

For Cygwin and OS X some specific setup has to be done prior to installation. These steps are described on the wiki pages of Bonmin [CygwinInstall](#)⁶ and [OsxInstall](#)⁷.

BONMIN is compiled and installed using the commands:

```
./configure -C
make
make install
```

This installs the executable `bonmin` in `Bonmin-1.7/bin`. In what follows, we assume that you have put the executable `bonmin` on your path.

The `configure` script attempts to find all of the machine specific settings (compiler, libraries,...) necessary to compile and run the code. Although `configure` should find most of the standard ones, you may have to manually specify a few of the settings. The options for the configure script can be found by issuing the command

```
./configure --help
```

⁶<https://projects.coin-or.org/Bonmin/wiki/CygwinInstall>

⁷<https://projects.coin-or.org/Bonmin/wiki/OsxInstall>

For a more in depth description of these options, the reader is invited to refer to the COIN-OR BuildTools [trac page](#)⁸.

3.1 Specifying the location of Cplex libraries

If you have Cplex installed on your machine, you may want to use it as the Mixed Integer Linear Programming subsolver in B-OA, B-Hyb and B-iFP. To do so you have to specify the location of the header files and libraries. You can either specify the location of the header files directory by passing it as an argument to the configure script or by writing it into a `config.site`.

In the former case, specify the location of the Cplex header files by using the argument `--with-cplexincdir` and the location of the Cplex library with `--with-cplexlib` (note that on the Linux platform you will also need to add `-lpthread -lm` as an argument to `--with-cplexlib`).

For example, on a Linux machine if Cplex is installed in `/usr/ilog`, you would invoke configure with the arguments as follows:

```
./configure --with-cplex-incdir=/usr/ilog/cplex/include/ilcplex \  
--with-cplex-lib="/usr/ilog/cplex/lib/libcplex.a -lpthread -lm"
```

In the latter case, put a file called `config.site` in a subdirectory named `share` of the installation directory (if you do not specify an alternate installation directory to the `configure` script with the `--prefix` argument, the installation directory is the directory where you execute the `configure` script). To specify the location of Cplex, insert the following lines in the `config.site` file:

```
with_cplex_lib="/usr/ilog/cplex/lib/libcplex.a -lpthread"  
with_cplex_incdir="/usr/ilog/cplex/include/ilcplex -lm"
```

(You will find a `config.site` example in the subdirectory `BuildTools` of `Bonmin-1.7`.)

3.2 Compiling BONMIN in an external directory

It is recommended to compile BONMIN in a directory different from the source directory (`Bonmin-1.7` in our case). This is convenient if you want to have several executables compiled for different architectures or have several executables compiled with different options (debugging and production, shared and static libraries) but also because you don't modify the directory where the sources are.

To do this just create a new directory, for example `Bonmin-build` in the parent directory of `Bonmin-1.7` and run configure from `Bonmin-build`:

```
../Bonmin-1.7/configure -C
```

This will create the makefiles in `Bonmin-build`, and you can then compile with the usual `make` and `make install` (in `Bonmin-build`).

⁸<https://projects.coin-or.org/BuildTools>

3.3 Building the documentation

The documentation for BONMIN consists of a users' manual (this document) and a reference manual. You can build a local copy of the reference manual provided that you have Latex and Doxygen installed on your machine. Issue the command `make doxydoc` in `Bonmin-1.7`. It calls Doxygen to build a copy of the reference manual. An html version of the reference manual can then be accessed in `doc/html/index.html`.

3.4 Running the test programs

By issuing the command `make test`, you build and run the automatic test program for BONMIN.

4 Running BONMIN

BONMIN can be run

- (i) from a command line on a `.nl` file (see [10]),
- (ii) from the modeling language `AMPL`⁹ (see [12]),
- (iii) from the `GAMS`¹⁰ modeling language,
- (iv) by invoking it from a C/C++ program,
- (v) remotely through the `NEOS`¹¹ web interface,
- (vi) via the `Optimization Services`¹² project,
- (vii) via the MATLAB `OPTI Toolbox`¹³.

In the subsections that follow, we give some details about the various ways to run BONMIN.

4.1 On a `.nl` file

BONMIN can read a `.nl` file which could be generated by `AMPL` (for example `mytoy.nl` in the `Bonmin-dist/Bonmin/test` subdirectory). The command line takes just one argument which is the name of the `.nl` file to be processed.

For example, if you want to solve `mytoy.nl`, from the `Bonmin-dist` directory, issue the command:

```
bonmin test/mytoy.nl
```

⁹<http://www.ampl.com>

¹⁰<http://www.gams.com/>

¹¹<http://neos.mcs.anl.gov/neos>

¹²<https://projects.coin-or.org/OS>

¹³<http://www.i2c2.aut.ac.nz/Wiki/OPTI>

4.2 From AMPL

To use BONMIN from AMPL you just need to have the directory where the `bonmin` executable is in your `$PATH` and to issue the command

```
option solver bonmin;
```

in the AMPL environment. Then the next `solve` will use BONMIN to solve the model loaded in AMPL. After the optimization is finished, the values of the variables in the best-known or optimal solution can be accessed in AMPL. If the optimization is interrupted with `<CTRL-C>` the best known solution is accessible (this feature is not available in Cygwin).

4.2.1 Example AMPL model

A simple AMPL example model follows:

```
# An AMPL version of toy

reset;

var x binary;
var z integer >= 0 <= 5;
var y{1..2} >=0;
minimize cost:
    - x - y[1] - y[2] ;

subject to
    c1: ( y[1] - 1/2 )^2 + (y[2] - 1/2)^2 <= 1/4 ;
    c2: x - y[1] <= 0 ;
    c3: x + y[2] + z <= 2;

option solver bonmin; # Choose BONMIN as the solver (assuming
                      # that bonmin is in your PATH)

solve;                # Solve the model
display x;
display y;
```

(This example model can be found in the BONMIN package in the subdirectory `Bonmin/examples/amplExamples/`.)

4.2.2 Setting up branching priorities, directions and declaring SOS1 constraints in AMPL

Branching priorities, branching directions and pseudo-costs can be passed using AMPL suffixes. The suffix for branching priorities is "priority" (variables with a higher priority will be chosen first for branching), for branching direction is "direction" (if direction is 1 the \geq branch is explored first, if direction is -1 the \leq branch is explored first), for up and down pseudo costs "upPseudoCost" and "downPseudoCost" respectively (note that if only one of the up and down pseudo-costs is set in the AMPL model it will be used for both up and down).

For example, to give branching priorities of 10 to variables y and 1 to variable x and to set the branching directions to explore the upper branch first for all variables in the simple example given, we add before the call to solve:

```
suffix priority IN, integer, >=0, <= 9999;
y[1].priority := 10;
y[2].priority := 10;
x.priority := 1;

suffix direction IN, integer, >=-1, <=1;
y[1].direction := 1;
y[2].direction := 1;
x.direction := 1;
```

SOS Type-1 branching is also available in BONMIN from AMPL. We follow the conventional way of doing this with suffixes. Two type of suffixes should be declared:

```
suffix sosno IN, integer, >=1; # Note that the solver assumes
                                # that these values are positive
                                # for SOS Type 1
suffix ref IN;
```

Next, suppose that we wish to have variables

```
var X {i in 1..M, j in 1..N} binary;
```

and the "convexity" constraints:

```
subject to Convexity {i in 1..M}:
    sum {j in 1..N} X[i,j] = 1;
```

(note that we must explicitly include the convexity constraints in the AMPL model).

Then after reading in the data, we set the suffix values:

```

# The numbers 'val[i,j]' are chosen typically as
#   the values 'represented' by the discrete choices.
let {i in 1..M, j in 1..N} X[i,j].ref := val[i,j];

# These identify which SOS constraint each variable belongs to.
let {i in 1..M, j in 1..N} X[i,j].sosno := i;

```

4.3 From GAMS

Thanks to the [GAMSLinks¹⁴](#) project, Bonmin is available in GAMS since release 22.5 of the [GAMS¹⁵](#) modeling system. The system is available for [download from GAMS¹⁶](#). Without buying a license it works as a demo with limited capabilities. Documentation for using BONMIN in GAMS is available at

<http://www.gams.com/solvers/coin.pdf>

4.4 From a C/C++ program

BONMIN can also be run from within a C/C++ program if the user codes the functions to compute first- and second-order derivatives. An example of such a program is available in the subdirectory `CppExample` of the `examples` directory. For further explanations, please refer to this example and to the reference manual.

5 Options

5.1 Passing options to BONMIN

Options in BONMIN can be set in several different ways.

First, you can set options by putting them in a file called `bonmin.opt` in the directory where `bonmin` is executing. If you are familiar with the file `ipopt.opt` (formerly named `PARAMS.DAT`) in `Ipopt`, the syntax of the `bonmin.opt` is similar. For those not familiar with `ipopt.opt`, the syntax is simply to put the name of the option followed by its value, with no more than two options on a single line. Anything on a line after a `#` symbol is ignored (i.e., treated as a comment).

Note that BONMIN sets options for `Ipopt`. If you want to set options for `Ipopt` (when used inside BONMIN) you have to set them in the file `bonmin.opt` (the standard `Ipopt` option file `ipopt.opt` is not read by BONMIN.) For a list and a description of all the `Ipopt` options, the reader may refer to the [documentation of Ipopt¹⁷](#).

¹⁴<http://projects.coin-or.org/GAMSLinks>

¹⁵<http://www.gams.com/>

¹⁶<http://download.gams.com/download>

¹⁷<http://www.coin-or.org/Ipopt/documentation/node39.html>

Since `bonmin.opt` contains both `Ipopt` and `BONMIN` options, for clarity all `BONMIN` options should be preceded with the prefix “`bonmin.`” in `bonmin.opt`. Note that some options can also be passed to the MILP subsolver used by `BONMIN` in the outer approximation decomposition and the hybrid (see Subsection 5.2).

The most important option in `BONMIN` is the choice of the solution algorithm. This can be set by using the option named `bonmin.algorithm` which can be set to `B-BB`, `B-OA`, `B-QG`, or `B-Hyb` (it’s default value is `B-BB`). Depending on the value of this option, certain other options may be available or not. Table 1 gives the list of options together with their types, default values and availability in each of the four algorithms. The column labeled ‘type’ indicates the type of the parameter (‘F’ stands for float, ‘I’ for integer, and ‘S’ for string). The column labeled ‘default’ indicates the global default value. Then for each of the algorithms `B-BB`, `B-OA`, `B-QG`, `B-Hyb`, `B-Ecp`, and `B-iFP` ‘√’ indicates that the option is available for that particular algorithm while ‘-’ indicates that it is not.

An example of a `bonmin.opt` file including all the options with their default values is located in the `Test` sub-directory.

A small example is as follows:

```
bonmin.bb_log_level 4
bonmin.algorithm B-BB
print_level 6
```

This sets the level of output of the branch-and-bound in `BONMIN` to 4, the algorithm to branch-and-bound and the output level for `Ipopt` to 6.

When `BONMIN` is run from within `AMPL`, another way to set an option is via the internal `AMPL` command `options`. For example

```
options bonmin_options "bonmin.bb_log-level 4 \
    bonmin.algorithm B-BB print_level 6";
```

has the same affect as the `bonmin.opt` example above. Note that any `BONMIN` option specified in the file `bonmin.opt` overrides any setting of that option from within `AMPL`.

A third way is to set options directly in the `C/C++` code when running `BONMIN` from inside a `C/C++` program as is explained in the reference manual.

A detailed description of all of the `BONMIN` options is given in Appendix A. In the following, we give some more details on options for the MILP subsolver and on the options specifically designed for nonconvex problems.

Table 1: List of options and compatibility with the different algorithms.

| Option | type | B-BB | B-OA | B-QG | B-Hyb | B-Ecp | B-iFP | Cbc_Par |
|---|------|------|------|------|-------|-------|-------|---------|
| Algorithm choice | | | | | | | | |
| algorithm | S | √ | √ | √ | √ | √ | √ | √ |
| Branch-and-bound options | | | | | | | | |
| allowable_fraction_gap | F | √ | √ | √ | √ | √ | √ | √ |
| allowable_gap | F | √ | √ | √ | √ | √ | √ | √ |
| cutoff | F | √ | √ | √ | √ | √ | √ | √ |
| cutoff_decr | F | √ | √ | √ | √ | √ | √ | √ |
| enable_dynamic_nlp | S | √ | - | - | - | - | - | - |
| integer_tolerance | F | √ | √ | √ | √ | √ | √ | √ |
| iteration_limit | I | √ | √ | √ | √ | √ | √ | √ |
| nlp_failure_behavior | S | √ | - | - | - | - | - | - |
| node_comparison | S | √ | √ | √ | √ | √ | √ | - |
| node_limit | I | √ | √ | √ | √ | √ | √ | √ |
| num_cut_passes | I | - | - | √ | √ | √ | - | - |
| num_cut_passes_at_root | I | - | - | √ | √ | √ | - | - |
| number_before_trust | I | √ | √ | √ | √ | √ | √ | √ |
| number_strong_branch | I | √ | √ | √ | √ | √ | √ | √ |
| random_generator_seed | I | √ | √ | √ | √ | √ | √ | √ |
| read_solution_file | S | √ | √ | √ | √ | √ | √ | √ |
| solution_limit | I | √ | √ | √ | √ | √ | √ | √ |
| sos_constraints | S | √ | √ | √ | √ | √ | √ | - |
| time_limit | F | √ | √ | √ | √ | √ | √ | √ |
| tree_search_strategy | S | √ | √ | √ | √ | √ | √ | - |
| variable_selection | S | √ | - | √ | √ | √ | - | - |
| ECP cuts generation | | | | | | | | |
| eep_abs_tol | F | - | - | √ | √ | - | - | - |
| eep_max_rounds | I | - | - | √ | √ | - | - | - |
| eep_probability_factor | F | - | - | √ | √ | - | - | - |
| eep_rel_tol | F | - | - | √ | √ | - | - | - |
| filmint_eep_cuts | I | - | - | √ | √ | - | - | - |
| Feasibility checker using OA cuts | | | | | | | | |
| feas_check_cut_types | S | - | - | √ | √ | √ | - | - |
| feas_check_discard_policy | S | - | - | √ | √ | √ | - | - |
| generate_benders_after_so_many_oa | I | - | - | √ | √ | √ | - | - |
| MILP Solver | | | | | | | | |
| cpx_parallel_strategy | I | - | - | - | - | - | - | √ |
| milp_solver | S | - | - | - | - | - | - | √ |
| milp_strategy | S | - | - | - | - | - | - | √ |
| number_cpx_threads | I | - | - | - | - | - | - | √ |
| MILP cutting planes in hybrid algorithm | | | | | | | | |
| continued on next page | | | | | | | | |

| Option | type | B-BB | B-OA | B-QG | B-Hyb | B-Ecp | B-iFP | Cbc_Par |
|--|------|------|------|------|-------|-------|-------|---------|
| 2mir_cuts | I | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Gomory_cuts | I | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| clique_cuts | I | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| cover_cuts | I | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| flow_cover_cuts | I | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| lift_and_project_cuts | I | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| mir_cuts | I | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| reduce_and_split_cuts | I | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| NLP interface | | | | | | | | |
| nlp_solver | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| warm_start | S | ✓ | - | - | - | - | - | - |
| NLP solution robustness | | | | | | | | |
| max_consecutive_failures | I | ✓ | - | - | - | - | - | - |
| max_random_point_radius | F | ✓ | - | - | - | - | - | - |
| num_iterations_suspect | I | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| num_retry_unsolved_random_point | I | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| random_point_perturbation_interval | F | ✓ | - | - | - | - | - | - |
| random_point_type | S | ✓ | - | - | - | - | - | - |
| resolve_on_small_infeasibility | F | ✓ | - | - | - | - | - | - |
| NLP solves in hybrid algorithm (B-Hyb) | | | | | | | | |
| nlp_solve_frequency | I | - | - | - | ✓ | - | - | - |
| nlp_solve_max_depth | I | - | - | - | ✓ | - | - | - |
| nlp_solves_per_depth | F | - | - | - | ✓ | - | - | - |
| Nonconvex problems | | | | | | | | |
| coeff_var_threshold | F | ✓ | - | - | - | - | - | - |
| dynamic_def_cutoff_decr | S | ✓ | - | - | - | - | - | - |
| first_perc_for_cutoff_decr | F | ✓ | - | - | - | - | - | - |
| max_consecutive_infeasible | I | ✓ | - | - | - | - | - | - |
| num_resolve_at_infeasibles | I | ✓ | - | - | - | - | - | - |
| num_resolve_at_node | I | ✓ | - | - | - | - | - | - |
| num_resolve_at_root | I | ✓ | - | - | - | - | - | - |
| second_perc_for_cutoff_decr | F | ✓ | - | - | - | - | - | - |
| Outer Approximation Decomposition (B-OA) | | | | | | | | |
| oa_decomposition | S | - | - | ✓ | ✓ | ✓ | - | - |
| Outer Approximation cuts generation | | | | | | | | |
| add_only_violated_oa | S | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| oa_cuts_scope | S | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| oa_rhs_relax | F | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| tiny_element | F | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| very_tiny_element | F | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Output and Loglevel | | | | | | | | |
| bb_log_interval | I | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| bb_log_level | I | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| continued on next page | | | | | | | | |

| Option | type | B-BB | B-OA | B-QG | B-Hyb | B-Ecp | B-iFP | Cbc_Par |
|--|------|------|------|------|-------|-------|-------|---------|
| file_solution | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| fp_log_frequency | F | - | - | ✓ | ✓ | - | - | - |
| fp_log_level | I | - | - | ✓ | ✓ | - | - | - |
| lp_log_level | I | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| milp_log_level | I | - | - | - | - | - | - | ✓ |
| nlp_log_at_root | I | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| nlp_log_level | I | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| oa_cuts_log_level | I | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| oa_log_frequency | F | ✓ | - | - | ✓ | ✓ | - | - |
| oa_log_level | I | ✓ | - | - | ✓ | ✓ | - | - |
| Primal Heuristics | | | | | | | | |
| feasibility_pump_objective_norm | I | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| fp_pass_infeasible | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| heuristic_RINS | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| heuristic_dive_MIP_fractional | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| heuristic_dive_MIP_vectorLength | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| heuristic_dive_fractional | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| heuristic_dive_vectorLength | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| heuristic_feasibility_pump | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| pump_for_minlp | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| Strong branching setup | | | | | | | | |
| candidate_sort_criterion | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| maxmin_crit_have_sol | F | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| maxmin_crit_no_sol | F | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| min_number_strong_branch | I | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| number_before_trust_list | I | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| number_look_ahead | I | ✓ | ✓ | ✓ | ✓ | ✓ | - | - |
| number_strong_branch_root | I | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| setup_pseudo_frac | F | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |
| trust_strong_branching_for_pseudo_cost | S | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - |

5.2 Passing options to local search based heuristics and oa generators

Several parts of the algorithms in BONMIN are based on solving a simplified version of the problem with another instance of BONMIN: Outer Approximation Decomposition (called in B-Hyb at the root node) and Feasibility Pump for MINLP (called in B-Hyb or B-BB at the root node), RINS, RENS, Local Branching.

In all these cases, one can pass options to the sub-algorithm used through the `bonmin.opt` file. The basic principle is that the `bonmin.` prefix is replaced with a prefix that identifies the sub-algorithm used:

- `oa_decomposition.` to pass options to Outer Approximation Decomposition,
- `pump_for_minlp.` to pass options to Feasibility Pump for MINLP
- `rins.` to pass options to RINS,
- `rens.` to pass options to RENS,
- `local_branch.` to pass options to Local Branching.

For example, we may want to run a maximum of 60 seconds of the feasibility pump for MINLP until 6 solutions are found at the beginning of the hybrid algorithm. To do so we set the following option in `bonmin.opt`

```
bonmin.algorithm B-Hyb
```

```
bonmin.pump_for_minlp yes      # tells to run fp for MINLP
pump_for_minlp.time_limit 60   # set a time limit for the pump
pump_for_minlp.solution_limit 6 # set a solution limit
```

Note that the actual solution and time limit will be the minimum of the global limits set for BONMIN.

A slightly more complicated set of options may be used when using RINS. Say for example that we want to run RINS inside B-BB. Each time RINS is called we want to solve the small-size MINLP generated using B-QG (we may run any algorithm available in BONMIN for solving an MINLP) and want to stop as soon as B-QG found 1 solution. We set the following options in `bonmin.opt`

```
bonmin.algorithm B-BB
```

```
bonmin.rins yes
rins.algorithm B-QG
rins.solution_limit 1
```

This example shows that it is possible to set any option used in the sub-algorithm to be different than the one used for the main algorithm.

In the context of OA and FP for MINLP, a standard MILP solver is used. Several options are available for configuring this MILP solver. `BONMIN` allows a choice of different MILP solvers through the option `bonmin.milp.subsolver`. Values for this option are: `Cbc_D` which uses `Cbc` with its default settings, `Cplex` which uses `Cplex` with its default settings, and `Cbc_Par` which uses a version of `Cbc` that can be parametrized by the user. The options that can be set in `Cbc_Par` are the number of strong-branching candidates, the number of branches before pseudo costs are to be trusted, and the frequency of the various cut generators (these options are signaled in Table 1).

5.3 Getting good solutions to nonconvex problems

To solve a problem with non-convex constraints, one should only use the branch-and-bound algorithm B-BB.

A few options have been designed in `BONMIN` specifically to treat problems that do not have a convex continuous relaxation. In such problems, the solutions obtained from `Ipopt` are not necessarily globally optimal, but are only locally optimal. Also the outer-approximation constraints are not necessarily valid inequalities for the problem.

No specific heuristic method for treating nonconvex problems is implemented yet within the OA framework. But for the pure branch-and-bound B-BB, we implemented a few options having in mind that lower bounds provided by `Ipopt` should not be trusted, and with the goal of trying to get good solutions. Such options are at a very experimental stage.

First, in the context of nonconvex problems, `Ipopt` may find different local optima when started from different starting points. The two options `num_resolve_at_root` and `num_resolve_at_node` allow for solving the root node or each node of the tree, respectively, with a user-specified number of different randomly-chosen starting points, saving the best solution found. Note that the function to generate a random starting point is very naïve: it chooses a random point (uniformly) between the bounds provided for the variable. In particular if there are some functions that can not be evaluated at some points of the domain, it may pick such points, and so it is not robust in that respect.

Secondly, since the solution given by `Ipopt` does not truly give a lower bound, we allow for changing the fathoming rule to continue branching even if the solution value to the current node is worse than the best-known solution. This is achieved by setting `allowable_gap` and `allowable_fraction_gap` and `cutoff_decr` to negative values.

5.4 Notes on Ipopt options

`Ipopt` has a very large number of options, to get a complete description of them, you should refer to the `Ipopt` manual. Here we only mention and explain some of the options that have been more important to us, so far, in developing and using `BONMIN`.

5.4.1 Default options changed by BONMIN

Ipopt has been tailored to be more efficient when used in the context of the solution of a MINLP problem. In particular, we have tried to improve **Ipopt**'s warm-starting capabilities and its ability to prove quickly that a subproblem is infeasible. For ordinary NLP problems, **Ipopt** does not use these options by default, but BONMIN automatically changes these options from their default values.

Note that options set by the user in `bonmin.opt` will override these settings.

`mu_strategy` and `mu_oracle` are set, respectively, to `adaptive` and `probing` by default (these are newly implemented strategies in **Ipopt** for updating the barrier parameter [13] which we have found to be more efficient in the context of MINLP).

`gamma_phi` and `gamma_theta` are set to 10^{-8} and 10^{-4} respectively. This has the effect of reducing the size of the filter in the line search performed by **Ipopt**.

`required_infeasibility_reduction` is set to 0.1. This increases the required infeasibility reduction when **Ipopt** enters the restoration phase and should thus help to detect infeasible problems faster.

`expect_infeasible_problem` is set to `yes`, which enables some heuristics to detect infeasible problems faster.

`warm_start_init_point` is set to `yes` when a full primal/dual starting point is available (generally all the optimizations after the continuous relaxation has been solved).

`print_level` is set to 0 by default to turn off **Ipopt** output.

5.4.2 Some useful **Ipopt** options

`bound_relax_factor` is by default set to 10^{-8} in **Ipopt**. All of the bounds of the problem are relaxed by this factor. This may cause some trouble when constraint functions can only be evaluated within their bounds. In such cases, this option should be set to 0.

References

- [1] K. Abhishek, S. Leyffer, and J. T. Linderoth. FilMINT: An outer-approximation-based solver for nonlinear mixed integer programs. *INFORMS Journal On Computing*, 22 (2010), pp. 555–567. doi:10.1287/ijoc.1090.0373

- [2] P. Bonami, A. Wächter, L.T. Biegler, A.R. Conn, G. Cornuéjols, I.E. Grossmann, C.D. Laird, J. Lee, A. Lodi, F. Margot and N. Sawaya. An algorithmic framework for convex mixed integer nonlinear programs. *Discrete Optimization*, 5 (2008), pp. 186–204. doi:10.1016/j.disopt.2006.10.011
- [3] P. Bonami, G. Cornuéjols, A. Lodi, and F. Margot. A feasibility pump for mixed integer nonlinear programs. *Mathematical Programming*, 119 (2009), pp. 331–352. doi:10.1007/s10107-008-0212-2
- [4] P. Bonami and J. Gonçalves. Primal heuristics for mixed integer nonlinear programs. *Computational Optimization and Applications*, 51 (2012), pp. 729–747. doi:10.1007/s10589-010-9350-6
- [5] P. Bonami, M. Kılınç and J. Linderoth. Algorithms and Software for Convex Mixed Integer Nonlinear Programs. In *Mixed Integer Nonlinear Programming* (J. Lee and S. Leyffer, editors), volume 154 of *The IMA Volumes in Mathematics and its Applications* (2012), pp. 1–40. doi:10.1007/978-1-4614-1927-3
- [6] M. Duran and I.E. Grossmann. An outer-approximation algorithm for a class of mixed-integer nonlinear programs. *Mathematical Programming*, 36 (1986), pp. 307–339. doi:10.1007/BF02592064
- [7] O.K. Gupta and V. Ravindran. Branch and bound experiments in convex nonlinear integer programming. *Management Science*, 31 (1985), pp. 1533–1546. doi:10.1287/mnsc.31.12.1533
- [8] R. Fletcher and S. Leyffer. Solving mixed integer nonlinear programs by outer approximation. *Mathematical Programming*, 66 (1994), pp. 327–349 doi:10.1007/BF01581153
- [9] R. Fletcher and S. Leyffer. User manual for filterSQP. *University of Dundee Numerical Analysis Report NA-181*, 1998.
- [10] D.M. Gay. Writing .nl files. Sandia National Laboratories, Technical Report No. 2005-7907P, 2005.
- [11] I. Quesada and I.E. Grossmann. An LP/NLP based branched and bound algorithm for convex MINLP optimization problems. *Computers and Chemical Engineering*, 16 (1992), pp. 937–947. doi:10.1016/0098-1354(92)80028-8
- [12] R. Fourer and D.M. Gay and B.W. Kernighan. AMPL: A Modeling Language for Mathematical Programming, Second Edition, Duxbury Press Brooks Cole Publishing Co., 2003.
- [13] J. Nocedal, A. Wächter, and R. A. Waltz. Adaptive Barrier Strategies for Nonlinear Interior Methods. *SIAM Journal on Optimization*, 19 (2008), pp. 1674–1693. doi:10.1137/060649513

- [14] A. Wächter and L. T. Biegler. On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Non-linear Programming. *Mathematical Programming* 106 (2006), pp. 25–57. [doi:10.1007/s10107-004-0559-y](https://doi.org/10.1007/s10107-004-0559-y)

A List of BONMIN options

A.1 Algorithm choice

algorithm: Choice of the algorithm.

This will preset some of the options of bonmin depending on the algorithm choice. The default value for this string option is "B-BB".

Possible values:

- B-BB: simple branch-and-bound algorithm,
- B-OA: OA Decomposition algorithm,
- B-QG: Quesada and Grossmann branch-and-cut algorithm,
- B-Hyb: hybrid outer approximation based branch-and-cut,
- B-Ecp: ECP cuts based branch-and-cut a la FilMINT.
- B-iFP: Iterated Feasibility Pump for MINLP.

A.2 Branch-and-bound options

allowable_fraction_gap: Specify the value of relative gap under which the algorithm stops.

Stop the tree search when the gap between the objective value of the best known solution and the best bound on the objective of any solution is less than this fraction of the absolute value of the best known solution value. The valid range for this real option is $-1 \cdot 10^{+20} \leq \text{allowable_fraction_gap} \leq 1 \cdot 10^{+20}$ and its default value is 0.

allowable_gap: Specify the value of absolute gap under which the algorithm stops.

Stop the tree search when the gap between the objective value of the best known solution and the best bound on the objective of any solution is less than this. The valid range for this real option is $-1 \cdot 10^{+20} \leq \text{allowable_gap} \leq 1 \cdot 10^{+20}$ and its default value is 0.

cutoff: Specify cutoff value.

cutoff should be the value of a feasible solution known by the user (if any). The algorithm will only look for solutions better than cutoff. The valid range for this real option is $-1 \cdot 10^{+100} \leq \text{cutoff} \leq 1 \cdot 10^{+100}$ and its default value is $1 \cdot 10^{+100}$.

cutoff_decr: Specify cutoff decrement.

Specify the amount by which cutoff is decremented below a new best upper-bound (usually a small positive value but in non-convex problems it may be a negative value). The valid range for this real option is $-1 \cdot 10^{+10} \leq \text{cutoff_decr} \leq 1 \cdot 10^{+10}$ and its default value is $1 \cdot 10^{-05}$.

enable_dynamic_nlp: Enable dynamic linear and quadratic rows addition in nlp

The default value for this string option is "no".

Possible values:

- no
- yes

integer_tolerance: Set integer tolerance.

Any number within that value of an integer is considered integer. The valid range for this real option is $0 < \text{integer_tolerance} < +\text{inf}$ and its default value is $1 \cdot 10^{-06}$.

iteration_limit: Set the cumulative maximum number of iteration in the algorithm used to process nodes continuous relaxations in the branch-and-bound. value 0 deactivates option. The valid range for this integer option is $0 \leq \text{iteration_limit} < +\text{inf}$ and its default value is 2147483647.

nlp_failure_behavior: Set the behavior when an NLP or a series of NLP are unsolved by Ipopt (we call unsolved an NLP for which Ipopt is not able to guarantee optimality within the specified tolerances).

If set to "fathom", the algorithm will fathom the node when Ipopt fails to find a solution to the nlp at that node within the specified tolerances. The algorithm then becomes a heuristic, and the user will be warned that the solution might not be optimal. The default value for this string option is "stop".

Possible values:

- stop: Stop when failure happens.
- fathom: Continue when failure happens.

node_comparison: Choose the node selection strategy.

Choose the strategy for selecting the next node to be processed. The default value for this string option is "best-bound".

Possible values:

- best-bound: choose node with the smallest bound,
- depth-first: Perform depth first search,
- breadth-first: Perform breadth first search,

- **dynamic:** Cbc dynamic strategy (starts with a depth first search and turn to best bound after 3 integer feasible solutions have been found).
- **best-guess:** choose node with smallest guessed integer solution

node_limit: Set the maximum number of nodes explored in the branch-and-bound search.

The valid range for this integer option is $0 \leq \text{node_limit} < +\text{inf}$ and its default value is 2147483647.

num_cut_passes: Set the maximum number of cut passes at regular nodes of the branch-and-cut.

The valid range for this integer option is $0 \leq \text{num_cut_passes} < +\text{inf}$ and its default value is 1.

num_cut_passes_at_root: Set the maximum number of cut passes at regular nodes of the branch-and-cut.

The valid range for this integer option is $0 \leq \text{num_cut_passes_at_root} < +\text{inf}$ and its default value is 20.

number_before_trust: Set the number of branches on a variable before its pseudo costs are to be believed in dynamic strong branching.

A value of 0 disables pseudo costs. The valid range for this integer option is $0 \leq \text{number_before_trust} < +\text{inf}$ and its default value is 8.

number_strong_branch: Choose the maximum number of variables considered for strong branching.

Set the number of variables on which to do strong branching. The valid range for this integer option is $0 \leq \text{number_strong_branch} < +\text{inf}$ and its default value is 20.

random_generator_seed: Set seed for random number generator (a value of -1 sets seeds to time since Epoch).

The valid range for this integer option is $-1 \leq \text{random_generator_seed} < +\text{inf}$ and its default value is 0.

read_solution_file: Read a file with the optimal solution to test if algorithms cuts it.

For Debugging purposes only. The default value for this string option is "no". Possible values:

- no
- yes

solution_limit: Abort after that much integer feasible solution have been found by algorithm
value 0 deactivates option The valid range for this integer option is $0 \leq \text{solution_limit} < +\text{inf}$ and its default value is 2147483647.

sos_constraints: Whether or not to activate SOS constraints.
(only type 1 SOS are supported at the moment) The default value for this string option is "enable".
Possible values:

- enable
- disable

time_limit: Set the global maximum computation time (in secs) for the algorithm.
The valid range for this real option is $0 \leq \text{time_limit} < +\text{inf}$ and its default value is $1 \cdot 10^{+10}$.

tree_search_strategy: Pick a strategy for traversing the tree
All strategies can be used in conjunction with any of the node comparison functions. Options which affect dfs-dive are max-backtracks-in-dive and max-dive-depth. The dfs-dive won't work in a non-convex problem where objective does not decrease down branches. The default value for this string option is "probed-dive".
Possible values:

- top-node: Always pick the top node as sorted by the node comparison function
- dive: Dive in the tree if possible, otherwise pick top node as sorted by the tree comparison function.
- probed-dive: Dive in the tree exploring two children before continuing the dive at each level.
- dfs-dive: Dive in the tree if possible doing a depth first search. Backtrack on leaves or when a prescribed depth is attained or when estimate of best possible integer feasible solution in subtree is worst than cutoff. Once a prescribed limit of backtracks is attained pick top node as sorted by the tree comparison function
- dfs-dive-dynamic: Same as dfs-dive but once enough solution are found switch to best-bound and if too many nodes switch to depth-first.

variable_selection: Chooses variable selection strategy
The default value for this string option is "strong-branching".
Possible values:

- most-fractional: Choose most fractional variable
- strong-branching: Perform strong branching
- reliability-branching: Use reliability branching
- qp-strong-branching: Perform strong branching with QP approximation
- lp-strong-branching: Perform strong branching with LP approximation
- nlp-strong-branching: Perform strong branching with NLP approximation
- osi-simple: Osi method to do simple branching
- osi-strong: Osi method to do strong branching
- random: Method to choose branching variable randomly

A.3 ECP cuts generation

ecp_abs_tol: Set the absolute termination tolerance for ECP rounds.
The valid range for this real option is $0 \leq \text{ecp_abs_tol} < +\text{inf}$ and its default value is $1 \cdot 10^{-06}$.

ecp_max_rounds: Set the maximal number of rounds of ECP cuts.
The valid range for this integer option is $0 \leq \text{ecp_max_rounds} < +\text{inf}$ and its default value is 5.

ecp_probability_factor: Factor appearing in formula for skipping ECP cuts.
Choosing -1 disables the skipping. The valid range for this real option is $-\text{inf} < \text{ecp_probability_factor} < +\text{inf}$ and its default value is 10.

ecp_rel_tol: Set the relative termination tolerance for ECP rounds.
The valid range for this real option is $0 \leq \text{ecp_rel_tol} < +\text{inf}$ and its default value is 0.

filmint_ecp_cuts: Specify the frequency (in terms of nodes) at which some a la filmint ecp cuts are generated.
A frequency of 0 amounts to to never solve the NLP relaxation. The valid range for this integer option is $0 \leq \text{filmint_ecp_cuts} < +\text{inf}$ and its default value is 0.

A.4 Feasibility checker using OA cuts

feas_check_cut_types: Choose the type of cuts generated when an integer feasible solution is found

If it seems too much memory is used should try Benders to use less The default value for this string option is "outer-approx".

Possible values:

- outer-approx: Generate a set of Outer Approximations cuts.
- Benders: Generate a single Benders cut.

feas_check_discard_policy: How cuts from feasibility checker are discarded Normally to avoid cycle cuts from feasibility checker should not be discarded in the node where they are generated. However Cbc sometimes does it if no care is taken which can lead to an infinite loop in Bonmin (usually on simple problems). To avoid this one can instruct Cbc to never discard a cut but if we do that for all cuts it can lead to memory problems. The default policy here is to detect cycles and only then impose to Cbc to keep the cut. The two other alternative are to instruct Cbc to keep all cuts or to just ignore the problem and hope for the best The default value for this string option is "detect-cycles".

Possible values:

- detect-cycles: Detect if a cycle occurs and only in this case force not to discard.
- keep-all: Force cuts from feasibility checker not to be discarded (memory hungry but sometimes better).
- treated-as-normal: Cuts from memory checker can be discarded as any other cuts (code may cycle then)

generate_benders_after_so_many_oa: Specify that after so many oa cuts have been generated Benders cuts should be generated instead.

It seems that sometimes generating too many oa cuts slows down the optimization compared to Benders due to the size of the LP. With this option we specify that after so many OA cuts have been generated we should switch to Benders cuts. The valid range for this integer option is $0 \leq \text{generate_benders_after_so_many_oa} < +\text{inf}$ and its default value is 5000.

A.5 MILP Solver

cpx_parallel_strategy: Strategy of parallel search mode in CPLEX.

-1 = opportunistic, 0 = automatic, 1 = deterministic (refer to CPLEX documentation) The valid range for this integer option is $-1 \leq \text{cpx_parallel_strategy} \leq 1$ and its default value is 0.

milp_solver: Choose the subsolver to solve MILP sub-problems in OA decompositions.

To use Cplex, a valid license is required and you should have compiled OsiCpx in COIN-OR (see Osi documentation). The default value for this string option is "Cbc_D".

Possible values:

- Cbc_D: Coin Branch and Cut with its default
- Cbc_Par: Coin Branch and Cut with passed parameters
- Cplex: IBM Cplex

milp_strategy: Choose a strategy for MILPs.

The default value for this string option is "solve_to_optimality".

Possible values:

- find_good_sol: Stop sub milps when a solution improving the incumbent is found
- solve_to_optimality: Solve MILPs to optimality

number_cpx_threads: Set number of threads to use with cplex.

(refer to CPLEX documentation) The valid range for this integer option is $0 \leq \text{number_cpx_threads} < +\text{inf}$ and its default value is 0.

A.6 MILP cutting planes in hybrid algorithm

2mir_cuts: Frequency (in terms of nodes) for generating 2-MIR cuts in branch-and-cut

If $k > 0$, cuts are generated every k nodes, if $-99 < k < 0$ cuts are generated every $-k$ nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if $k = -99$ generate cuts only at the root node, if $k = 0$ or 100 do not generate cuts. The valid range for this integer option is $-100 \leq \text{2mir_cuts} < +\text{inf}$ and its default value is 0.

Gomory_cuts: Frequency (in terms of nodes) for generating Gomory cuts in branch-and-cut.

If $k > 0$, cuts are generated every k nodes, if $-99 < k < 0$ cuts are generated every $-k$ nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if $k = -99$ generate cuts only at the root node, if $k = 0$ or 100 do not generate cuts. The valid range for this integer option is $-100 \leq \text{Gomory_cuts} < +\text{inf}$ and its default value is -5 .

clique_cuts: Frequency (in terms of nodes) for generating clique cuts in branch-and-cut

If $k > 0$, cuts are generated every k nodes, if $-99 < k < 0$ cuts are generated every $-k$ nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if $k = -99$ generate cuts only at the root node, if $k = 0$ or 100 do not generate cuts. The valid range for this integer option is $-100 \leq \text{clique_cuts} < +\text{inf}$ and its default value is -5 .

cover_cuts: Frequency (in terms of nodes) for generating cover cuts in branch-and-cut

If $k > 0$, cuts are generated every k nodes, if $-99 < k < 0$ cuts are generated every $-k$ nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if $k = -99$ generate cuts only at the root node, if $k = 0$ or 100 do not generate cuts. The valid range for this integer option is $-100 \leq \text{cover_cuts} < +\text{inf}$ and its default value is 0 .

flow_cover_cuts: Frequency (in terms of nodes) for generating flow cover cuts in branch-and-cut

If $k > 0$, cuts are generated every k nodes, if $-99 < k < 0$ cuts are generated every $-k$ nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if $k = -99$ generate cuts only at the root node, if $k = 0$ or 100 do not generate cuts. The valid range for this integer option is $-100 \leq \text{flow_cover_cuts} < +\text{inf}$ and its default value is -5 .

lift_and_project_cuts: Frequency (in terms of nodes) for generating lift-and-project cuts in branch-and-cut

If $k > 0$, cuts are generated every k nodes, if $-99 < k < 0$ cuts are generated every $-k$ nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if $k = -99$ generate cuts only at the root node, if $k = 0$ or 100 do not generate cuts. The valid range for this integer option is $-100 \leq \text{lift_and_project_cuts} < +\text{inf}$ and its default value is 0 .

mir_cuts: Frequency (in terms of nodes) for generating MIR cuts in branch-and-cut

If $k > 0$, cuts are generated every k nodes, if $-99 < k < 0$ cuts are generated every $-k$ nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if $k = -99$ generate cuts only at the root node, if $k = 0$ or 100 do not generate cuts. The valid range for this integer option is $-100 \leq \text{mir_cuts} < +\text{inf}$ and its default value is -5 .

reduce_and_split_cuts: Frequency (in terms of nodes) for generating reduce-and-split cuts in branch-and-cut

If $k > 0$, cuts are generated every k nodes, if $-99 < k < 0$ cuts are generated every $-k$ nodes but Cbc may decide to stop generating cuts, if not enough are generated at the root node, if $k = -99$ generate cuts only at the root node, if

$k = 0$ or 100 do not generate cuts. The valid range for this integer option is $-100 \leq \text{reduce_and_split_cuts} < +\text{inf}$ and its default value is 0 .

A.7 NLP interface

nlp_solver: Choice of the solver for local optima of continuous NLP's

Note that option will work only if the specified solver has been installed. Ipopt will usually be installed with Bonmin by default. For FilterSQP please see <http://www-unix.mcs.anl.gov/leyffer/solvers.html> on how to obtain it and <https://projects.coin-or.org/Bonmin/wiki/HintTricks> on how to configure Bonmin to use it. The default value for this string option is "Ipopt".

Possible values:

- Ipopt: Interior Point OPTimizer (<https://projects.coin-or.org/Ipopt>)
- filterSQP: Sequential quadratic programming trust region algorithm (<http://www-unix.mcs.anl.gov/leyffer/solvers.html>)
- all: run all available solvers at each node

warm_start: Select the warm start method

This will affect the function `getWarmStart()`, and as a consequence the warm starting in the various algorithms. The default value for this string option is "none".

Possible values:

- none: No warm start, just start NLPs from optimal solution of the root relaxation
- fake_basis: builds fake basis, useful for cut management in Cbc (warm start is the same as in none)
- optimum: Warm start with direct parent optimum
- interior_point: Warm start with an interior point of direct parent

A.8 NLP solution robustness

max_consecutive_failures: (temporarily removed) Number n of consecutive unsolved problems before aborting a branch of the tree.

When $n > 0$, continue exploring a branch of the tree until n consecutive problems in the branch are unsolved (we call unsolved a problem for which Ipopt can not guarantee optimality within the specified tolerances). The valid range for this integer option is $0 \leq \text{max_consecutive_failures} < +\text{inf}$ and its default value is 10 .

max_random_point_radius: Set max value r for coordinate of a random point.

When picking a random point, coordinate i will be in the interval $[\min(\max(l, -r), u-r), \max(\min(u, r), l+r)]$ (where l is the lower bound for the variable and u is its upper bound) The valid range for this real option is $0 < \text{max_random_point_radius} < +\text{inf}$ and its default value is 100000.

num_iterations_suspect: Number of iterations over which a node is considered "suspect" (for debugging purposes only, see detailed documentation).

When the number of iterations to solve a node is above this number, the sub-problem at this node is considered to be suspect and it will be written into a file (set to -1 to deactivate this). The valid range for this integer option is $-1 \leq \text{num_iterations_suspect} < +\text{inf}$ and its default value is -1.

num_retry_unsolved_random_point: Number k of times that the algorithm will try to resolve an unsolved NLP with a random starting point (we call unsolved an NLP for which Ipopt is not able to guarantee optimality within the specified tolerances).

When Ipopt fails to solve a continuous NLP sub-problem, if $k > 0$, the algorithm will try again to solve the failed NLP with k new randomly chosen starting points or until the problem is solved with success. The valid range for this integer option is $0 \leq \text{num_retry_unsolved_random_point} < +\text{inf}$ and its default value is 0.

random_point_perturbation_interval: Amount by which starting point is perturbed when choosing to pick random point by perturbing starting point The valid range for this real option is $0 < \text{random_point_perturbation_interval} < +\text{inf}$ and its default value is 1.

random_point_type: method to choose a random starting point

The default value for this string option is "Jon".

Possible values:

- Jon: Choose random point uniformly between the bounds
- Andreas: perturb the starting point of the problem within a prescribed interval
- Claudia: perturb the starting point using the perturbation radius suffix information

resolve_on_small_infeasibility: If a locally infeasible problem is infeasible by less than this, resolve it with initial starting point.

It is set to 0 by default with Ipopt. When using FilterSQP, Bonmin sets it to a small value. The valid range for this real option is $0 \leq \text{resolve_on_small_infeasibility} < +\text{inf}$ and its default value is 0.

A.9 NLP solves in hybrid algorithm (B-Hyb)

nlp_solve_frequency: Specify the frequency (in terms of nodes) at which NLP relaxations are solved in B-Hyb.

A frequency of 0 amounts to to never solve the NLP relaxation. The valid range for this integer option is $0 \leq \text{nlp_solve_frequency} < +\text{inf}$ and its default value is 10.

nlp_solve_max_depth: Set maximum depth in the tree at which NLP relaxations are solved in B-Hyb.

A depth of 0 amounts to to never solve the NLP relaxation. The valid range for this integer option is $0 \leq \text{nlp_solve_max_depth} < +\text{inf}$ and its default value is 10.

nlp_solves_per_depth: Set average number of nodes in the tree at which NLP relaxations are solved in B-Hyb for each depth.

The valid range for this real option is $0 \leq \text{nlp_solves_per_depth} < +\text{inf}$ and its default value is $1 \cdot 10^{+100}$.

A.10 Nonconvex problems

coeff_var_threshold: Coefficient of variation threshold (for dynamic definition of cutoff_decr).

The valid range for this real option is $0 \leq \text{coeff_var_threshold} < +\text{inf}$ and its default value is 0.1.

dynamic_def_cutoff_decr: Do you want to define the parameter cutoff_decr dynamically?

The default value for this string option is "no".

Possible values:

- no
- yes

first_perc_for_cutoff_decr: The percentage used when, the coeff of variance is smaller than the threshold, to compute the cutoff_decr dynamically.

The valid range for this real option is $-\text{inf} < \text{first_perc_for_cutoff_decr} < +\text{inf}$ and its default value is -0.02 .

max_consecutive_infeasible: Number of consecutive infeasible subproblems before aborting a branch.

Will continue exploring a branch of the tree until "max_consecutive_infeasible" consecutive problems are locally infeasible by the NLP sub-solver. The valid range for this integer option is $0 \leq \text{max_consecutive_infeasible} < +\text{inf}$ and its default value is 0.

num_resolve_at_infeasibles: Number k of tries to resolve an infeasible node (other than the root) of the tree with different starting point.

The algorithm will solve all the infeasible nodes with k different random starting points and will keep the best local optimum found. The valid range for this integer option is $0 \leq \text{num_resolve_at_infeasibles} < +\text{inf}$ and its default value is 0.

num_resolve_at_node: Number k of tries to resolve a node (other than the root) of the tree with different starting point.

The algorithm will solve all the nodes with k different random starting points and will keep the best local optimum found. The valid range for this integer option is $0 \leq \text{num_resolve_at_node} < +\text{inf}$ and its default value is 0.

num_resolve_at_root: Number k of tries to resolve the root node with different starting points.

The algorithm will solve the root node with k random starting points and will keep the best local optimum found. The valid range for this integer option is $0 \leq \text{num_resolve_at_root} < +\text{inf}$ and its default value is 0.

second_perc_for_cutoff_decr: The percentage used when, the coeff of variance is greater than the threshold, to compute the cutoff_decr dynamically.

The valid range for this real option is $-\text{inf} < \text{second_perc_for_cutoff_decr} < +\text{inf}$ and its default value is -0.05 .

A.11 Outer Approximation Decomposition (B-OA)

oa_decomposition: If yes do initial OA decomposition

The default value for this string option is "no".

Possible values:

- no
- yes

A.12 Outer Approximation cuts generation

add_only_violated_oa: Do we add all OA cuts or only the ones violated by current point?

The default value for this string option is "no".

Possible values:

- no: Add all cuts
- yes: Add only violated cuts

oa_cuts_scope: Specify if OA cuts added are to be set globally or locally valid
The default value for this string option is "global".

Possible values:

- local: Cuts are treated as locally valid
- global: Cuts are treated as globally valid

oa_rhs_relax: Value by which to relax OA cut
RHS of OA constraints will be relaxed by this amount times the absolute value of the initial rhs if it is $\epsilon = 1$ (otherwise by this amount). The valid range for this real option is $-0 \leq \text{oa_rhs_relax} < +\text{inf}$ and its default value is $1 \cdot 10^{-08}$.

tiny_element: Value for tiny element in OA cut
We will remove "cleanly" (by relaxing cut) an element lower than this. The valid range for this real option is $-0 \leq \text{tiny_element} < +\text{inf}$ and its default value is $1 \cdot 10^{-08}$.

very_tiny_element: Value for very tiny element in OA cut
Algorithm will take the risk of neglecting an element lower than this. The valid range for this real option is $-0 \leq \text{very_tiny_element} < +\text{inf}$ and its default value is $1 \cdot 10^{-17}$.

A.13 Output and Loglevel

bb_log_interval: Interval at which node level output is printed.
Set the interval (in terms of number of nodes) at which a log on node resolutions (consisting of lower and upper bounds) is given. The valid range for this integer option is $0 \leq \text{bb_log_interval} < +\text{inf}$ and its default value is 100.

bb_log_level: specify main branch-and-bound log level.
Set the level of output of the branch-and-bound : 0 - none, 1 - minimal, 2 - normal low, 3 - normal high The valid range for this integer option is $0 \leq \text{bb_log_level} \leq 5$ and its default value is 1.

file_solution: Write a file bonmin.sol with the solution
The default value for this string option is "no".
Possible values:

- yes
- no

fp_log_frequency: display an update on lower and upper bounds in FP every n seconds
The valid range for this real option is $0 < \text{fp_log_frequency} < +\text{inf}$ and its default value is 100.

fp_log_level: specify FP iterations log level.

Set the level of output of OA decomposition solver : 0 - none, 1 - normal, 2 - verbose The valid range for this integer option is $0 \leq \text{fp_log_level} \leq 2$ and its default value is 1.

lp_log_level: specify LP log level.

Set the level of output of the linear programming sub-solver in B-Hyb or B-QG : 0 - none, 1 - minimal, 2 - normal low, 3 - normal high, 4 - verbose The valid range for this integer option is $0 \leq \text{lp_log_level} \leq 4$ and its default value is 0.

milp_log_level: specify MILP solver log level.

Set the level of output of the MILP subsolver in OA : 0 - none, 1 - minimal, 2 - normal low, 3 - normal high The valid range for this integer option is $0 \leq \text{milp_log_level} \leq 4$ and its default value is 0.

nlp_log_at_root: specify a different log level for root relaxation.

The valid range for this integer option is $0 \leq \text{nlp_log_at_root} \leq 12$ and its default value is 0.

nlp_log_level: specify NLP solver interface log level (independent from ipopt print_level).

Set the level of output of the OsiTMINLPInterface : 0 - none, 1 - normal, 2 - verbose The valid range for this integer option is $0 \leq \text{nlp_log_level} \leq 2$ and its default value is 1.

oa_cuts_log_level: level of log when generating OA cuts.

0: outputs nothing, 1: when a cut is generated, its violation and index of row from which it originates, 2: always output violation of the cut. 3: output generated cuts incidence vectors. The valid range for this integer option is $0 \leq \text{oa_cuts_log_level} < +\text{inf}$ and its default value is 0.

oa_log_frequency: display an update on lower and upper bounds in OA every n seconds

The valid range for this real option is $0 < \text{oa_log_frequency} < +\text{inf}$ and its default value is 100.

oa_log_level: specify OA iterations log level.

Set the level of output of OA decomposition solver : 0 - none, 1 - normal, 2 - verbose The valid range for this integer option is $0 \leq \text{oa_log_level} \leq 2$ and its default value is 1.

A.14 Primal Heuristics

feasibility_pump_objective_norm: Norm of feasibility pump objective function

The valid range for this integer option is $1 \leq \text{feasibility_pump_objective_norm} \leq 2$ and its default value is 1.

fp_pass_infeasible: Say whether feasibility pump should claim to converge or not

The default value for this string option is "no".

Possible values:

- no: When master MILP is infeasible just bail out (don't stop all algorithm). This is the option for using in B-Hyb.
- yes: Claim convergence, numerically dangerous.

heuristic_RINS: if yes runs the RINS heuristic

The default value for this string option is "no".

Possible values:

- no
- yes

heuristic_dive_MIP_fractional: if yes runs the Dive MIP Fractional heuristic

The default value for this string option is "no".

Possible values:

- no
- yes

heuristic_dive_MIP_vectorLength: if yes runs the Dive MIP VectorLength heuristic

The default value for this string option is "no".

Possible values:

- no
- yes

heuristic_dive_fractional: if yes runs the Dive Fractional heuristic

The default value for this string option is "no".

Possible values:

- no
- yes

heuristic_dive_vectorLength: if yes runs the Dive VectorLength heuristic
The default value for this string option is "no".

Possible values:

- no
- yes

heuristic_feasibility_pump: whether the heuristic feasibility pump should
be used

The default value for this string option is "no".

Possible values:

- no
- yes

pump_for_minlp: whether to run the feasibility pump heuristic for MINLP

The default value for this string option is "no".

Possible values:

- no
- yes

A.15 Strong branching setup

candidate_sort_criterion: Choice of the criterion to choose candidates in
strong-branching

The default value for this string option is "best-ps-cost".

Possible values:

- best-ps-cost: Sort by decreasing pseudo-cost
- worst-ps-cost: Sort by increasing pseudo-cost
- most-fractional: Sort by decreasing integer infeasibility
- least-fractional: Sort by increasing integer infeasibility

maxmin_crit_have_sol: Weight towards minimum in of lower and upper branch-
ing estimates when a solution has been found.

The valid range for this real option is $0 \leq \text{maxmin_crit_have_sol} \leq 1$ and its
default value is 0.1.

maxmin_crit_no_sol: Weight towards minimum in of lower and upper branch-
ing estimates when no solution has been found yet.

The valid range for this real option is $0 \leq \text{maxmin_crit_no_sol} \leq 1$ and its
default value is 0.7.

min_number_strong_branch: Sets minimum number of variables for strong branching (overriding trust)

The valid range for this integer option is $0 \leq \text{min_number_strong_branch} < +\text{inf}$ and its default value is 0.

number_before_trust_list: Set the number of branches on a variable before its pseudo costs are to be believed during setup of strong branching candidate list.

The default value is that of "number.before.trust" The valid range for this integer option is $-1 \leq \text{number_before_trust_list} < +\text{inf}$ and its default value is 0.

number_look_ahead: Sets limit of look-ahead strong-branching trials

The valid range for this integer option is $0 \leq \text{number_look_ahead} < +\text{inf}$ and its default value is 0.

number_strong_branch_root: Maximum number of variables considered for strong branching in root node.

The valid range for this integer option is $0 \leq \text{number_strong_branch_root} < +\text{inf}$ and its default value is 2147483647.

setup_pseudo_frac: Proportion of strong branching list that has to be taken from most-integer-infeasible list.

The valid range for this real option is $0 \leq \text{setup_pseudo_frac} \leq 1$ and its default value is 0.5.

trust_strong_branching_for_pseudo_cost: Whether or not to trust strong branching results for updating pseudo costs.

The default value for this string option is "yes".

Possible values:

- no
- yes